1.
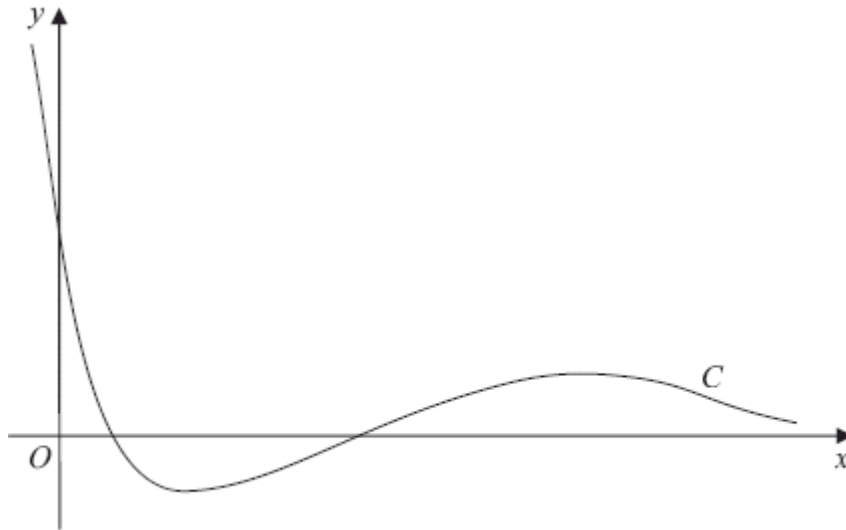

The diagram above shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a)   Find the coordinates of the point where C crosses the y-axis.

(1)

(b)   Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis.

(3)

(c)   Find $\dfrac{dy}{dx}$.

(3)

(d)   Hence find the exact coordinates of the turning points of C.

(5)
(Total 12 marks)

2.   (i)   Given that $y = \dfrac{\ln(x^2 + 1)}{x}$, find $\dfrac{dy}{dx}$.

(4)

(ii)   Given that $x = \tan y$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{1 + x^2}$.

(5)
(Total 9 marks)

3.   (a)   By writing $\sec x$ as $\dfrac{1}{\cos x}$, show that $\dfrac{\mathrm{d}(\sec x)}{\mathrm{d}x} = \sec x \tan x$.

(3)

Given that $y = e^{2x} \sec 3x$,

(b)   find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

The curve with equation $y = e^{2x} \sec 3x,\ -\dfrac{\pi}{6} < x < \dfrac{\pi}{6},$ has a minimum turning point at (a, b).

(c)   Find the values of the constants a and b, giving your answers to 3 significant figures.

(4)
(Total 11 marks)

4.   (i)   Differentiate with respect to x

(a) $x^2 \cos 3x$

(3)

(b) $\dfrac{\ln(x^2 + 1)}{x^2 + 1}$

(4)

(ii)   A curve C has the equation

$$y = \sqrt{(4x + 1)}, \, x > -\frac{1}{4}, \, y > 0$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)
(Total 13 marks)

5.   The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R}, \, x \neq -4, \, x \neq 2$$

(a)   Show that $f(x) = \dfrac{x-3}{x-2}$

(5)

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \qquad x \in \mathbb{R}, x \neq \ln 2$$

(b)   Differentiate g(x) to show that $g'(x) = \dfrac{e^x}{(e^x - 2)^2}$,

(3)

(c)   Find the exact values of x for which $g'(x) = 1$

(4)
(Total 12 marks)

6.   (a)   Find the value of $\dfrac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2 \sqrt{(5x-1)}.$$

(6)

(b)   Differentiate $\dfrac{\sin 2x}{x^2}$ with respect to x.

(4)
(Total 10 marks)

7.
$$f(x) = \frac{2x+2}{x^2 - 2x - 3} - \frac{x+1}{x-3}$$

(a)   Express f (x) as a single fraction in its simplest form.

(4)

(b)   Hence show that $f'(x) = \dfrac{2}{(x-3)^2}$

(3)
(Total 7 marks)

8.
$$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P.

(a)   Find the exact coordinates of P.

(5)

The equation f $(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b)   Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.

(3)

(c)   By choosing a suitable interval, show that a root of f(x) = 0 is x = 0.2576 correct to 4
       decimal places.

(3)
(Total 11 marks)

9.   (a)   Differentiate with respect to x,

(i) $e^{3x}(\sin x + 2\cos x)$,

(3)

(ii) $x^3 \ln (5x + 2)$.

(3)

Given that $y = \dfrac{3x^2 + 6x - 7}{(x+1)^2}, \ x \neq -1,$

(b)   show that $\dfrac{dy}{dx} = \dfrac{20}{(x+1)^3}$.

(5)

(c)   Hence find $\dfrac{d^2y}{dx^2}$ and the real values of x for which $\dfrac{d^2y}{dx^2} = -\dfrac{15}{4}$.

(3)
(Total 14 marks)

10.   A curve C has equation

$$y = e^{2x} \tan x, \ x \neq (2n+1)\frac{\pi}{2}$$

(a)   Show that the turning points on C occur where $\tan x = -1$.

(6)

(b)   Find an equation of the tangent to C at the point where $x = 0$.

(2)
(Total 8 marks)

11. $$f(x) = (x^2 + 1) \ln x, \quad x > 0.$$

    (a)   Use differentiation to find the value of f′(x) at $x = $ e, leaving your answer in terms of e.

                                                 (4)

    (b)   Find the exact value of $\displaystyle\int_1^e f(x)\,dx$

                                                 (5)
                                (Total 9 marks)

12.   (a)   Differentiate with respect to x

         (i) $x^2 e^{3x+2}$,

                                                 (4)

         (ii) $\dfrac{\cos(2x^3)}{3x}$.

                                                 (4)

    (b)   Given that $x = 4\sin(2y + 6)$, find $\dfrac{dy}{dx}$ in terms of x.

                                                 (5)
                                (Total 13 marks)

13.   (a)   Differentiate with respect to x

         (i) $3\sin^2 x + \sec 2x$,

                                                 (3)

         (ii) $\{x + \ln(2x)\}^3$.

                                                 (3)

    Given that $y = \dfrac{5x^2 - 10x + 9}{(x-1)^2}, \ x \neq 1$,

    (b)   show that $\dfrac{dy}{dx} = -\dfrac{8}{(x-1)^3}$.

                                                 (6)
                                (Total 12 marks)

14.   Differentiate with respect to x

(i) $x^3 \, e^{3x}$,

(3)

(ii) $\dfrac{2x}{\cos x}$,

(3)

(iii) $\tan^2 x$.

(2)

Given that $x = \cos y^2$,

(iv)   find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y.

(4)
(Total 12 marks)

1. (a) Either $y = 2$ or $(0, 2)$ B1   1

(b) When $x = 2$, $y = (8 - 10 + 2)e^{-2} = 0e^{-2} = 0$ B1

  $(2x^2 - 5x + 2) = 0 \implies (x - 2)(2x - 1) = 0$

  Either $x = 2$ (for possibly B1 above) or $\underline{x = \frac{1}{2}}$.   A1   3

  Note

  If the candidate believes that $e^{-x} = 0$ solves to $x = 0$ or gives an extra
  solution of $x = 0$, then withhold the final accuracy mark.

(c) $\dfrac{dy}{dx} = (4x - 5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$ A1 A1   3

  Note

      (their u')e^{-x} + $(2x^2 - 5x + 2)$(their v')
  A1: Any one term correct.
  A1: Both terms correct.

(d) $(4x - 5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0$

  $2x^2 - 9x + 7 = 0 \Rightarrow (2x - 7)(x - 1) = 0$

  $x = \frac{7}{2}, 1$ A1

  When $x = \frac{7}{2}$, $y = 9e^{-\frac{7}{2}}$, when $x = 1$, $y = -e^{-1}$ dd   A1   5

  Note

  1st For setting their $\frac{dy}{dx}$ found in part (c) equal to 0.

  2nd Factorise or eliminate out e^{-x} correctly and an attempt to factorise
  a 3-term quadratic or apply the formula to candidate's $ax^2 + bx + c$.
  See rules for solving a three term quadratic equation on page 1 of
  this Appendix.
  3rd ddM1: An attempt to use at least one x-coordinate on $y = (2x^2 - 5x + 2)e^{-x}$.
  Note that this method mark is dependent on the award of the two
  previous method marks in this part.
  Some candidates write down corresponding y-coordinates without any
  working. It may be necessary on some occasions to use your calculator
  to check that at least one of the two y-coordinates found is correct
  to awrt 2 sf.

  Final A1: Both$\{x = 1\}$, $y = -e^{-1}$ and $\left\{x = \frac{7}{2}\right\}$, $y = 9e^{-\frac{7}{2}}$. **cao**
  Note that both exact values of y are required.

                                                                                                         [12]

2.   (i) $y = \dfrac{\ln(x^2 + 1)}{x}$

$u = \ln(x^2 + 1) \Rightarrow \dfrac{du}{dx} = \dfrac{2x}{x^2 + 1}$ $\ln(x^2 + 1) \to \dfrac{\text{something}}{x^2 + 1}$

                                                                    $\ln(x^2 + 1) \to \dfrac{2x}{x^2 + 1}$ A1

Apply quotient rule:

$\begin{Bmatrix} u = \ln(x^2 + 1) & v = x \\ \dfrac{du}{dx} = \dfrac{2x}{x^2 + 1} & \dfrac{dv}{dx} = 1 \end{Bmatrix}$

$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{2x}{x^2 + 1}\right)(x) - \ln(x^2 + 1)}{x^2}$ Applying $\dfrac{xu' - \ln(x^2 + 1)v'}{x^2}$

                                                    correctly.

                                                    Correct differentiation
                                                    with correct bracketing
                                                    but allow recovery.   A1   4

$\left\{ \dfrac{dy}{dx} = \dfrac{2}{(x^2 + 1)} - \dfrac{1}{x^2}\ln(x^2 + 1) \right\}$ {Ignore subsequent working.}

(ii) $x = \tan y$ $\tan y \to \sec^2 y$ or an attempt to

$\dfrac{dx}{dy} = \sec^2 y$ differentiate $\dfrac{\sin y}{\cos y}$

                                    using either the   *

                        quotient rule or product rule.

                                                    $\dfrac{dx}{dy} = \sec^2 y$ A1

$\dfrac{dx}{dy} = \dfrac{1}{\sec^2 y}\{= \cos^2 y\}$ Finding $\dfrac{dy}{dx}$ by reciprocating $\dfrac{dx}{dy}$ dM1 *

$\dfrac{dy}{dx} = \dfrac{1}{1 + \tan^2 y}$ For writing down or

                                    applying the identity

                                    $\sec^2 y = 1 + \tan^2 y$,   dM1 *

                        which must be applied/stated
                                    completely in *y*.

Hence, $\dfrac{dy}{dx} = \dfrac{1}{1 + x^2}$,

(as required)   For the correct proof, leading
                        on from the previous line of
                                    working.   A1 **AG**   5

[9]

3.   (a) $y = \sec x = \dfrac{1}{\cos x}$

$= (\cos x)^{-1}$

$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$ $\dfrac{dy}{dx} = \pm 1((\cos x)^{-2}(-\sin x))$

$-1(\cos x)^{-2}(-\sin x)$ or
$(\cos x)^{-2}(\sin x)$ A1

$\dfrac{dy}{dx} = \left\{ \dfrac{\sin x}{\cos^2 x} \right\} =$

$\underline{\underline{\left(\dfrac{1}{\cos x} \right)\left(\dfrac{\sin x}{\cos x} \right)}} = \underline{\underline{\sec x \tan x}}$ Convincing proof.

Must see both
underline steps .   A1 **AG**   3

(b) $y = e^{2x}\sec 3x$

$\begin{cases} u = e^{2x} & v = \sec 3x \\ \dfrac{du}{dx} = 2e^{2x} & \dfrac{dv}{dx} = 3\sec 3x \tan 3x \end{cases}$ $\boxed{\begin{array}{c} \text{Seen} \\ \text{or implied} \end{array}}$

Either $e^{2x} \to 2e^{2x}$ or
$\sec 3x \to 3\sec 3x \tan 3x$

Both $e^{2x} \to 2e^{2x}$ and
$\sec 3x \to 3\sec 3x \tan 3x$ A1

$\dfrac{dy}{dx} = 2e^{2x}\sec 3x + 3e^{2x}\sec 3x$
$\tan 3x$ Applies $vu' + uv'$ correctly
for their u, u', v, v'

$2e^{2x}\sec 3x + 3e^{2x}\sec 3x\tan 3x$ A1 **isw**   4

(c)   Turning point $\Rightarrow \dfrac{dy}{dx} = 0$

Hence, $e^{2x}\sec 3x(2 + 3\tan 3x) = 0$ Sets their $\dfrac{dy}{dx} = 0$ and
factorises (or cancels)

$\{$Note $e^{2x} \neq 0$, $\sec 3x \neq 0$, so
$2 + 3\tan 3x = 0,\}$ out at least e^{2x} from at
least two terms.

giving $\tan 3x = -\frac{2}{3}$ $\tan 3x = \pm k$; $k \neq 0$

$\Rightarrow 3x = -0.58800 \Rightarrow x = \{a\}$
$= -0.19600...$ Either awrt -0.196^c or
                                                                awrt $-11.2°$ A1

Hence, $y = \{b\} = e^{2(-0.196)}$ sec
(3×-0.196)

$\quad = 0.812093... =$
$\quad 0.812$ (3sf)   0.812   A1 **cao**   4

Note

If there are any EXTRA solutions for x (or a)
inside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$ or ANY
EXTRA solutions for y (or b), (for these values of x) then
withhold the final accuracy mark.

Also ignore EXTRA solutions outside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$,
ie. $-0.524 < x < 0.524$.

[11]

4.   (i)   (a) $y = x^2 \cos 3x$

Apply product rule: $\begin{cases} u = x^2 & v = \cos 3x \\ \dfrac{du}{dx} = 2x & \dfrac{dv}{dx} = -3\sin 3x \end{cases}$

Applies $vu' + uv'$ correctly for
their u, u', v, v' AND gives an
expression of the form
$\alpha\, x \cos 3x \pm \beta\, x^2 \sin 3x$

$\dfrac{dy}{dx} = 2x\cos 3x - 3x^2 \sin 3x$ Any one term correct   A1

Both terms correct and no further
simplification to terms in   A1   3
$\cos\alpha\, x^2$ or $\sin\beta\, x^3$.

(b) $y = \dfrac{\ln(x^2 + 1)}{x^2 + 1}$

$u = \ln(x^2 + 1) \Rightarrow \dfrac{du}{dx} = \dfrac{2x}{x^2 + 1}$ $\ln(x^2 + 1) \rightarrow \dfrac{\text{something}}{x^2 + 1}$

$\ln(x^2 + 1) \rightarrow \dfrac{2x}{x^2 + 1}$ A1

Apply quotient rule: $\begin{cases} u = \ln(x^2 + 1) & v = x^2 + 1 \\ \dfrac{du}{dx} = \dfrac{2x}{x^2 + 1} & \dfrac{dv}{dx} = 2x \end{cases}$

$$\frac{dy}{dx} = \frac{\left(\dfrac{2x}{x^2+1}\right)(x^2+1) - 2x\ln(x^2+1)}{(x^2+1)^2}$$ Applying $\dfrac{vu' - uv'}{v^2}$

Correct differentiation with correct   A1   4
bracketing but allow recovery.

$$\left\{\frac{dy}{dx} = \frac{2x - 2x\ln(x^2+1)}{(x^2+1)^2}\right\}$$ {Ignore subsequent working.}

(ii) $y = \sqrt{4x+1},\ x > -\frac{1}{4}$

At P, $y = \sqrt{4(2)+1} = \underline{\sqrt{9}} = \underline{3}$ At P, $y = \underline{\sqrt{9}}$ or $\underline{3}$ B1

$$\frac{dy}{dx} = \underline{\frac{1}{2}(4x+1)^{-\frac{1}{2}}}$$ $\pm k(4x+1)^{-\frac{1}{2}}$ *

                                                        $2(4x+1)^{-\frac{1}{2}}$ A1 aef

$$\frac{dy}{dx} = \frac{2}{(4x+1)^{\frac{1}{2}}}$$

At P, $\dfrac{dy}{dx} = \dfrac{2}{(4(2)+1)^{\frac{1}{2}}}$ Substituting $x = 2$ into an equation

                                                        involving $\frac{dy}{dx}$;

Hence m(**T**) = $\dfrac{2}{3}$

                                                        $y - y_1 = m(x - 2)$

Either **T**: $y - 3 = \frac{2}{3}(x-2)$;   or 1 $y - y_1 = m(x -$ their stated x) with
                                                        'their TANGENT gradient' and
or $y = \frac{2}{3}x + c$ and   their y_1;   dM1 * ;
$3 = \frac{2}{3}(2) + c \Rightarrow c = 3 - \frac{4}{3} = \frac{5}{3}$;   or uses $y = mx + c$ with
                                                        'their TANGENT gradient', their x
                                                        and their y_1.

Either **T**: $3y - 9 = 2(x - 2)$;

T: $3y - 9 = 2x - 4$

T: $\underline{2x - 3y + 5 = 0}$ $\underline{2x - 3y + 5 = 0}$ A1   6
                                                Tangent must be stated in the form
                                                $ax + by + c = 0$, where a, b and c
                                                        are integers.

or **T**: $y = \frac{2}{3}x + \frac{5}{3}$

T: $3y = 2x + 5$

T: $\underline{2x - 3y + 5 = 0}$

                                                                                [13]

5.   (a) $f(x) = 1 - \dfrac{2}{(x+4)} + \dfrac{x-8}{(x-2)(x+4)}$

$x \in \mathbb{R}, x \neq -4, x \neq 2.$

$f(x) = \dfrac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$ An attempt to combine

to one fraction
Correct result of combining all   A1
three fractions

$= \dfrac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$

$= \dfrac{x^2 + x - 12}{[(x+4)(x-2)]}$ Simplifies to give the correct

numerator. Ignore omission of denominator   A1

$= \dfrac{(x+4)(x-3)}{[(x+4)(x-2)]}$ An attempt to factorise the   dM1

numerator.

$= \dfrac{(x-3)}{(x-2)}$ Correct result   A1 cso **AG**   5

(b) $g(x) = \dfrac{e^x - 3}{e^x - 2}$ $x \in \mathbb{R}, x \neq \ln 2.$

Apply quotient rule: $\begin{cases} u = e^x - 3 & v = e^x - 2 \\ \dfrac{du}{dx} = e^x & \dfrac{dv}{dx} = e^x \end{cases}$

$g'(x) = \dfrac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$ Applying $\dfrac{vu' - uv'}{v^2}$

Correct differentiation   A1

$= \dfrac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2}$

$= \dfrac{e^x}{(e^x - 2)^2}$ Correct result   A1 **AG**

cso   3

(c) g'(x) = 1 $\Rightarrow$ $= \dfrac{e^x}{(e^x - 2)^2} = 1$

$e^x = (e^x - 2)^2$ Puts their differentiated numerator
                                   equal to their denominator.

$e^x = e^{2x} - 2e^x - 2e^x + 4$

$\underline{e^{2x} - 5e^x + 4 = 0}$ $\underline{e^{2x} - 5e^x + 4}$ A1

$(e^x - 4)(e^x - 1) = 0$ Attempt to factorise
                                   or solve quadratic in e^x

$e^x = 4$ or $e^x = 1$

$x = \ln 4$ or $x = 0$ both $x = 0$, $\ln 4$ A1   4

[12]

6.   (a) $\dfrac{d}{dx}(\sqrt{(5x-1)}) = \dfrac{d}{dx}\left((5x-1)^{\frac{1}{2}}\right)$

$\qquad\qquad\qquad\quad = 5 \times \dfrac{1}{2}(5x-1)^{-\frac{1}{2}}$ A1

$\dfrac{d}{dx} = 2x\sqrt{(5x-1)} + \dfrac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$ A1ft

At $x = 2$, $\dfrac{dy}{dx} = 4\sqrt{9} + \dfrac{10}{\sqrt{9}} = 12 + \dfrac{10}{3}$

$\qquad\quad = \dfrac{46}{3}$ Accept awrt 15.3   A1   6

(b) $\dfrac{d}{dx}\left(\dfrac{\sin 2x}{x^2}\right) = \dfrac{2x^2\cos 2x - 2x\sin 2x}{x^4}$ $\dfrac{A1 + A1}{A1}$ 4

Alternative

$\dfrac{d}{dx}(\sin 2x \times x^{-2}) = 2\cos 2x \times x^{-2} + \sin 2x \times (-2)x^{-3}$ A1 + A1

$2x^{-2}\cos 2x - 2x^{-3}\sin 2x = \left(\dfrac{2\cos 2x}{x^2} - \dfrac{2\sin 2x}{x^3}\right)$ A1   4

[10]

7. (a)
$$\frac{2x+2}{x^2-2x-3}-\frac{x+1}{x-3}=\frac{2x+2}{(x-3)(x+1)}-\frac{x+1}{x-3}$$

$$=\frac{2x+2-(x+1)(x+1)}{(x-3)(x+1)} \qquad \text{A1}$$

$$=\frac{(x+1)(1-x)}{(x-3)(x+1)}$$

$$\frac{1-x}{x-3} \qquad \text{Accept} - \frac{x-1}{x-3},\frac{x-1}{3-x} \qquad \text{A1} \quad 4$$

Alternative

$$\frac{2x+2}{x^2-2x-3}=\frac{2(x+1)}{(x-3)(x+1)}=\frac{2}{x-3} \qquad \text{A1}$$

$$\frac{2}{x-3}-\frac{x+1}{x-3}=\frac{2-(x+1)}{x-3}$$

$$\frac{1-x}{x-3} \qquad \text{A1} \quad 4$$

(b)
$$\frac{\text{d}}{\text{d}x}\left(\frac{1-x}{x-3}\right)=\frac{(x-3)(-1)-(1-x)1}{(x-3)^2} \qquad \text{A1}$$

$$=\frac{-x+3-1+x}{(x-3)^2}=\frac{2}{(x-3)^2} \qquad * \qquad \text{cso} \qquad \text{A1} \quad 3$$

Alternatives

① $$\text{f}(x)=\frac{1-x}{x-3}=-1-\frac{2}{x-3}=-1-2(x-3)^{-1}$$

$$\text{f'}(x)=(-1)(-2)(x-3)^{-2} \qquad \text{A1}$$

$$=\frac{2}{(x-3)^2} \qquad * \qquad \text{cso} \qquad \text{A1} \quad 3$$

② $$\text{f}(x)=(1-x)(x-3)^{-1}$$

$$\text{f'}(x)=(-1)(x-3)^{-1}+(1-x)(-1)(x-3)^{-2}$$

$$=-\frac{1}{x-3}-\frac{1-x}{(x-3)^2}=\frac{-(x-3)-(1-x)}{(x-3)^2} \qquad \text{A1}$$

$$=\frac{2}{(x-3)^2} \qquad * \qquad \text{A1} \quad 3$$

[7]

8. (a) $$f'(x) = 3e^x + 3xe^x \qquad \text{A1}$$

$$3e^x + 3xe^x = 3ex\,(1+x) = 0$$

$$x = -1 \qquad \text{M1A1}$$

$$f(-1) = -3e^{-1} - 1 \qquad \text{B1} \qquad 5$$

(b) $$x_1 = 0.2596 \qquad \text{B1}$$

$$x_2 = 0.2571 \qquad \text{B1}$$

$$x_3 = 0.2578 \qquad \text{B1} \qquad 3$$

(c) Choosing (0.257 55, 0.257 65) or an appropriate tighter interval.

$$f(0.257\,55) = -0.000\,379\,...$$

$$f(0.257\,65) = 0.000109\,… \qquad \text{A1}$$

Change of sign (and continuity) $\Rightarrow$ root $\in (0.257\,55, 0.257\,65)$ $*$ cso A1 3

$(\Rightarrow x = 0.2576$, is correct to 4 decimal places)

Note:

x = 0.257 627 65 ... is accurate

[11]

9. (a) (i) $$\frac{\mathrm{d}}{\mathrm{d}x}(e^{3x}(\sin x + 2\cos x)) = 3e^{3x}(\sin x + 2\cos x) + e^{3x}(\cos x - 2\sin x)$$

$$(= e^{3x}(\sin x + 7\cos x)) \qquad \text{M1A1A1} \qquad 3$$

(ii) $$\frac{\mathrm{d}}{\mathrm{d}x}(x^3 \ln(5x+2)) = 3x^2 \ln(5x+2) + \frac{5x^3}{5x+2} \qquad \text{M1A1A1} \qquad 3$$

(b) $$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4} \qquad \text{M1}\,\genfrac{}{}{0pt}{}{\text{A1}}{\text{A1}}$$

$$= \frac{(x+1)(6x^2+12x+6-6x^2-12x+14)}{(x+1)^4}$$

$$= \frac{20}{(x+1)^3} * \qquad \text{cso} \qquad \text{A1} \qquad 5$$

Note: The simplification can be carried out as follows

$$= \frac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$$

$$= \frac{(6x^3 + 18x^2 + 18x + 6) - (6x^3 + 18x^2 - 2x - 14)}{(x+1)^4}$$

$$= \frac{20x + 20}{(x+1)^4} = \frac{20(x+1)}{(x+1)^4} = \frac{20}{(x+1)^3} \qquad \text{A1}$$

(c) $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\dfrac{60}{(x+1)^4} = -\dfrac{15}{4}$

$(x+1)^4 = 16$

$x = 1, -3$ both   A1   3

[14]

10. (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2e^{2x} \tan x + e^{2x} \sec^2 x$ A1+A1

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow 2e^{2x} \tan x + e^{2x} \sec^2 x = 0$

$2\tan x + 1 + \tan^2 x = 0$ A1

$(\tan x + 1)^2 = 0$

$\tan x = -1$ (*)   cso   A1   6

(b) $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = 1$

Equation of tangent at (0, 0) is $y = x$ A1   2

[8]

11. (a) $f'(x) = (x^2 + 1) \times \dfrac{1}{x} + \ln x \times 2x$ A1

$f'(e) = (e + 1) \times \dfrac{1}{e} + 2e = 3e + \dfrac{1}{e}$ A1   4

(b) $(\dfrac{x^3}{3} + x)\ln x - \int (\dfrac{x^3}{3} + x)\dfrac{1}{x} dx$ A1

$= (\dfrac{x^3}{3} + x)\ln x - \int (\dfrac{x^3}{3} + 1) dx$

$= \left[(\dfrac{x^3}{3} + x)\ln x - (\dfrac{x^3}{9} + x)\right]_1^e$ A1

$= \dfrac{2}{9}e^3 + \dfrac{10}{9}$ A1   5

[9]

12. (a) (i) $\frac{d}{dx}(e^{3x+2}) = 3e^{3x+2}$ (or $3e^2e^{3x}$)   At any stage   B1

  $\frac{dy}{dx} = 3x^2\,e^{3x+2} + 2xe^{3x+2}$ Or equivalent   A1+A1   4

(ii) $\frac{d}{dx}(\cos(2x^3)) = -6x^2\sin(2x^3)$ At any stage   A1

  $\frac{dy}{dx} = \frac{-18x^3\sin(2x^3) - 3\cos(2x^3)}{9x^2}$ A1   4

Alternatively using the product rule for second   A1

$$y = (3x)^{-1}\cos(2x^3)$$

$$\frac{dy}{dx} = -3(3x)^{-2}\cos(2x^3) - 6x^2(3x)^{-1}\sin(2x^3)$$

Accept equivalent unsimplified forms

(b) $1 = 8\cos(2y+6)\frac{dy}{dx}$ or $\frac{dx}{dy} = 8\cos(2y+6)$

  $\frac{dy}{dx} = \frac{1}{8\cos(2y+6)}$ A1

  $\frac{dy}{dx} = \frac{1}{8\cos\left(\arcsin\left(\frac{x}{4}\right)\right)}$ $\left(= (\pm)\frac{1}{2\sqrt{(16-x^2)}}\right)$ A1   5

[13]

13. (a) (i) $6\sin x\cos x + 2\sec 2x\tan 2x$ A1 A1   3
      or $3\sin 2x + 2\sec 2x\tan 1x$
        [for 6 sinx]

(ii) $3(x + \ln 2x)^2(1 + \frac{1}{x})$ B1   A1   3

    [B1 for $3(x + \ln 2x)^2$]

(b)   Differentiating numerator to obtain $10x - 10$ B1
      Differentiating denominator to obtain $2(x - 1)$ B1

Using quotient rule formula correctly:

To obtain $\frac{dy}{dx} = \frac{(x-1)^2(10x-10) - (5x^2 - 10x + 9)(2(x-1))}{(x-1)^4}$ A1

Simplifying to form $\frac{2(x-1)[5(x-1)^2 - (5x^2 - 10x + 9)]}{(x-1)^4}$

$$= -\frac{8}{(x-1)^3} \quad (*)$$ c.s.o. A1   6

Alternatives for (b)
Either Using product rule formula correctly:
Obtaining $10x - 10$ B1
Obtaining $-2(x-1)^{-3}$

To obtain $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (5x^2 - 10x + 9)\{-2(x-1)^{-3}\} + (10x - 10)(x-1)^{-2}$ A1cao

Simplifying to form $\dfrac{10(x-1)^2 - 2(5x^2 - 10x + 9)}{(x-1)^3}$

$$= -\frac{8}{(x-1)^3} \quad (*)$$ c.s.o. A1   6

Or Splitting fraction to give $5 + \dfrac{4}{(x-1)^2}$ B1 B1

Then differentiating to give answer   A1 A1   6

                                                                         [12]

14.   (i) $u = x^3 \quad \dfrac{\mathrm{d}u}{\mathrm{d}x} = 3x^2$

$v = \mathrm{e}^{3x} \quad \dfrac{\mathrm{d}v}{\mathrm{d}x} = 3\mathrm{e}^{3x}$

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3x^2\,\mathrm{e}^{3x} + x^3 3\mathrm{e}^{3x}$ or equiv   A1 A1   3

   (ii) $u = 2x \quad \dfrac{\mathrm{d}u}{\mathrm{d}x} = 2$

$v = \cos x \quad \dfrac{\mathrm{d}v}{\mathrm{d}x} = -\sin x$

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2\cos x + 2x\sin x}{\cos^2 x}$ or equiv   A1 A1   3

   (iii) $u = \tan x \qquad \dfrac{\mathrm{d}u}{\mathrm{d}x} = \sec^2 x$

$y = u^2 \qquad \dfrac{\mathrm{d}y}{\mathrm{d}u} = 2u$

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2u\,\sec^2 x$

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\tan x\,\sec^2 x$ A1   2

(iv) $u = y^2$ $\dfrac{\mathrm{d}u}{\mathrm{d}y} = 2y$

$x = \cos u$ $\dfrac{\mathrm{d}x}{\mathrm{d}u} = -\sin u$

$\dfrac{\mathrm{d}x}{\mathrm{d}y} = -2y \sin y^2$ A1

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-1}{2y \sin y^2}$ A1   4

[12]

1. This question was extremely well answered with 84% of candidates gaining at least 7 of the 12 marks available and about 42% gaining all 12 marks.

Nearly all candidates were successful in answering part (a). A few candidates were initially confused when attempting part (a) by believing that the curve met the *y*-axis when $y = 0$. These candidates quickly recovered and relabelled part (a) as their part (b) and then went onto to find in part (a) that when $x = 0$, $y = 2$. Therefore, for these candidates, part (b) was completed before part (a).

In part (b), some candidates chose to substitute $x = 2$ into $y = (2x^2 - 5x + 2)\mathrm{e}^{-x}$ in order to confirm that $y = 0$ The majority of candidates, however, set $y = 0$ and solved the resulting equation to give both $x = 2$ and $x = 0.5$ Only a few candidates wrote that $x = 0$ is a solution of $\mathrm{e}^{-x} = 0$.

In part (c), the product rule was applied correctly to $(2x^2 - 5x + 2)\mathrm{e}^{-x}$ by a very high proportion of candidates with some simplifying the result to give $(-2x^2 + 9x - 7)\mathrm{e}^{-x}$ Common errors included either e^{-x} being differentiated incorrectly to give e^{-x} or poor bracketing. The quotient rule was rarely seen, but when it was it was usually applied correctly.

In part (d), the majority of candidates set their $\dfrac{dy}{dx}$ in part (c) equal to 0, although a few differentiated again and set $\dfrac{d^2 y}{dx^2} = 0$. At this stage, few candidates produced invalid logarithmic work and lost a number of marks. Some other candidates made bracketing and/or algebraic errors in simplifying their gradient function. Most candidates realised that they needed to factorise out e^{-x} and solve the resulting quadratic with many of them correctly finding both sets of coordinates. Some candidates did not give their *y*-coordinates in terms of e, but instead wrote the decimal equivalent.

2.   In part (i), the quotient rule was generally well applied in most candidates' working, although those candidates who decided to use the product rule in this part were usually successful in gaining all 4 marks.

A significant number of candidates struggled to differentiate $\ln(x^2 + 1)$ correctly. $\dfrac{1}{x^2 + 1}$, $\dfrac{2}{x^2 + 1}$ or even $\dfrac{1}{x}$ were common incorrect outcomes. In this part, candidates were not required to simplify their differentiated result and a significant number of them continued to simplify their answer further having gained all 4 marks. A significant number of these candidates appeared to struggle here owing to their weak algebraic and manipulative skills.

Candidates found part (ii) more demanding. Many candidates were able to write down $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ correctly in terms of y and understood the process of taking the reciprocal to find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. Some candidates wrote $x = \tan y$ as $x = \dfrac{\sin y}{\cos y}$ and used the quotient rule to differentiate the result. At this point, some candidates did not make the link with the differentiated $\sec^2 y$ and $x = \tan y$ given in the question. Some candidates quoted the identity $\sec^2 y = 1 + \tan^2 y$ in the wrong variable and so it was not possible for them to complete the proof and score the final 2 marks in this part. There were a significant number of candidates who wrote $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ as $\sec^2 x$, and so failed to score any marks for this part.

A large number of candidates, however, tried to make a link between part (i) and part (ii) of this question. These candidates either substituted $x = \tan y$ into their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ expression from part (i) or substituted $x = \tan y$ into $y = \dfrac{\ln\left(x^2 + 1\right)}{x}$. Some of these candidates then wasted time by unsuccessfully trying to prove the required result. It appeared that the more proficient candidates avoided this pitfall.

3.   In part (a), candidates used the quotient rule more often than a direct chain rule. The quotient rule was often spoiled by some candidates who wrote down that 1 was the derivative of 1. Another common error was for candidates to write down that $0 \times \cos x = \cos x$. Some proofs missed out steps and only fully convincing proofs gained all 3 marks with the final mark sometimes lost through lack of an explicit demonstration.

In part (b), many candidates differentiated e^{2x} correctly but common mistakes for the derivative of $\sec 3x$ were $\sec 3x \tan 3x$ or $3\sec x \tan x$. The product rule was applied correctly to $\mathrm{e}^{2x}\sec 3x$ by a very high proportion of candidates, although occasionally some candidates applied the quotient rule to differentiate $\dfrac{\mathrm{e}^2 x}{\sec 3x}$. A few candidates applied the quotient rule to differentiate $\mathrm{e}^{2x}\sec 3x$ when the product rule would have been correct.

Those candidates, who attempted to differentiate $e^{2x}\sec 3x$ in part (b), were able to set their $\dfrac{dy}{dx}$ equal to zero and factorise out at least e^{2x}, with a significant number of candidates getting as far as $\tan 3x = \pm k$, $(k \neq 0)$, with some candidates giving k as -2. Many of the candidates who achieved $k = -\frac{2}{3}$, were able to find the correct answer for a of -0.196, although a few of them incorrectly stated a as 0.196. A surprising number of good candidates, having found the correct value for x, were then unable to correctly evaluate y. It was not required to prove the nature of the turning point, so it was a waste of several candidates' time to find an expression for the second derivative.

4.   Part (i)(a) was well answered by the majority of candidates. The most common error was incorrectly differentiating $\cos 3x$ to either $3\sin 3x$ or $-\sin 3x$. A few candidates lost the final accuracy mark for simplification errors such as simplifying $(\cos 3x)\,(2x)$ to $\cos 6x^2$.

In (i)(b), the quotient rule was generally well applied in most candidates' working. A significant number of candidates, however, struggled to differentiate $\ln(x^2 + 1)$ correctly.

$\dfrac{1}{x^2+1}, \dfrac{2}{x^2+1}$ or even $\dfrac{1}{x}$ were common incorrect outcomes. Those candidates who decided to use the product rule in this part were less successful in gaining some or all of the marks.

Again part (ii) was generally well attempted by candidates of all abilities. The most common error was incorrectly differentiating $\sqrt{(4x+1)}$ although a few candidates failed to attempt to differentiate this. A few candidates found the equation of the normal and usually lost the final two marks. Also, a number of candidates failed to write the equation of the tangent in the correct form and so lost the final accuracy mark.

5.   Many candidates were able to obtain the correct answer in part (a) with a significant number of candidates making more than one attempt to arrive at the answer given in the question. Those candidates who attempted to combine all three terms at once or those who combined the first two terms and then combined the result with the third term were more successful in this part. Other candidates who started by trying to combine the second and third terms had problems dealing with the negative sign in front $\dfrac{2}{x+4}$ and usually added $\dfrac{2}{(x+4)}$ to $\dfrac{x-8}{(x-2)(x+4)}$ before combining the result with 1. It was pleasing to see that very few candidates used $(x+4)^2(x-2)$ as their common denominator when combining all three terms.

In part (b), most candidates were able to apply the quotient rule correctly but a number of candidates failed to use brackets properly in the numerator and then found some difficultly in arriving at the given answer.

In part (c), many candidates were able to equate the numerator to the denominator of the given fraction and many of these candidates went onto form a quadratic in e^x which they usually solved. A significant number of candidates either failed to spot the quadratic or expanded $(e^x - 2)^2$ and then took the natural logarithm of each term on both sides of their resulting equation.

In either or both of parts (b) and (c), some candidates wrote e^{x^2} in their working instead of e^{2x} Such candidates usually lost the final accuracy mark in part (b) and the first accuracy mark

in part (c).

6. This proved a good starting question which tested the basic laws of differentiation; the chain, product and quotient laws. Almost all candidates were able to gain marks on the question. In part (a), most realised that they needed to write $\sqrt{(5x-1)}$ as $(5x-1)^{\frac{1}{2}}$ before differentiating. The commonest error was to give $\frac{d}{dx}\left((5x-1)^{\frac{1}{2}}\right) = \frac{1}{2}(5x-1)^{-\frac{1}{2}}$, omitting the factor 5. It was disappointing to see a number of candidates incorrectly interpreting brackets, writing $(5x-1)^{\frac{1}{2}} = 5x^{\frac{1}{2}} - 1^{\frac{1}{2}}$. Not all candidates realised that the product rule was needed and the use of $\frac{d}{dx}(uv) = \frac{du}{dx} \times \frac{dv}{dx}$ was not uncommon. Part (b) was generally well done but candidates should be aware of the advantages of starting by quoting a correct quotient rule. The examiner can then award method marks even if the details are incorrect. The commonest error seen was writing $\frac{d}{dx}$ $(\sin 2x) = \cos 2x$. A number of candidates caused themselves unnecessary difficulties by writing $\sin 2x = 2\sin x \cos x$. Those who used the product rule in part (b) seemed, in general, to be more successful than those who had used this method in other recent examinations.

7. This type of question has been set quite frequently and the majority of candidates knew the method well. Most approached the question in the conventional way by expressing the fractions with the common denominator $(x - 3)(x + 1)$. This question can, however, be made simpler by cancelling down the first fraction by $(x + 1)$, obtaining $\frac{2x+2}{x^2-2x-3} = \frac{2(x+1)}{(x+3)(x+1)} = \frac{2}{x-3}$.

    Those who used the commoner method often had difficulties with the numerator of the combined fraction, not recognising that $-x^2 + 1 = 1 - x^2 = (1 - x)(1 + x)$ can be used to simplify this fraction. If part (a) was completed correctly, part (b) was almost invariably correct. It was possible to gain full marks in part (b) from unsimplified fractions in part (a), but this was rarely achieved.

8. A substantial proportion of candidates did not recognise that, in part (a), the product rule is needed to differentiate $3x\,e^x$ and $3x^2\,e^x$, $3x\,e^x$ and $3e^x$ were all commonly seen. It was also not uncommon for the question to be misinterpreted and for $3x\,(e^x - 1)$ to be differentiated. Those who did differentiate correctly usually completed part (a) correctly. Part (b) was very well done with the majority of the candidates gaining full marks. Very few lost marks for truncating their decimals or giving too many decimal places.

    In parts (c), candidates need to be aware that showing that something is true requires them to give reasons and conclusions. It would be sufficient to argue that a change of sign in the interval(0.257 55, 0.257 65)implies that there is a root in the interval (0.257 55, 0.257 65) and, hence, that $x = 0.2576$ is correct to 4 decimal places. The majority of candidates did provide an acceptable argument. Fewer candidates than usual attempted repeated iteration, an method that is explicitly ruled out by the wording of the question.

9.   This was a very discriminating question and candidates who obtained full marks and very few marks were both quite common. When using the product or quotient rules, candidates should be encouraged to quote the rules as it is often very difficult for examiners to establish whether or not candidates are attempting a correct method. In part (a), many made errors differentiating e^{3x}, $\sin x$ and, especially, $\ln (5x + 2)$. In part (b), a lack of bracketing, again, lead to confusion. On using the quotient rule, many candidates did not notice the factor $(x + 1)$ at the first stage and went on to expand two cubic expressions in the numerator. This caused extra work but the majority who used this method did complete the question correctly.

In part (d), those who wrote $\dfrac{dy}{dx}$ as $20 (x + 1)$ and differentiated directly, usually completed the question quickly. However many used the quotient rule and, for many, this caused great difficulty. It was not uncommon to see $\dfrac{d}{dx}(20) = 1$ written down and many, who avoided this mistake directly, nevertheless produced a numerator which implied that $(x + 1)^3 \times 0 = (x + 1)^3$. This lead to an equation which is not solvable at this level and frequently much time was wasted in a fruitless attempt to solve it.

10.   In part (a), the majority of candidates were able to handle the differentiation competently and most were aware that their result had to be equated to zero. The subsequent work in part (a) was less well done with relatively few candidates completing the proof. Of those candidates who recognised the need to use the identity of $\sec^2 x = 1 + \tan^2 x$, many were unsure what to do with the e^x factor. A common approach to completing this part of the question was to attempt a verification of $\tan x = -1$ by substituting one specific value, $x = -45°$ or $x = -\dfrac{\pi}{4}$, into their differentiated expression. This gained a maximum of 4 out of the 6 marks, as the examiners wished to see the general result established.

In part (b), candidates that had found $\dfrac{dy}{dx}$ correctly in part (b) were usually able to find the gradient and proceed to a correct equation. However, a significant number had their tangent passing through $(0,1)$ to give an equation of $y = x + 1$. A minority thought that the result $\tan x = -1$ in part (a) implied that the gradient in part (b) was -1.

11.   The product rule was well understood and many candidates correctly differentiated f(x) in part (a). However, a significant number lost marks by failing to use $\ln e = 1$ and fully simplify their answer.

Although candidates knew that integration by parts was required for part (b), the method was not well understood with common wrong answers involving candidates mistakenly suggesting that $\displaystyle\int \ln x \, dx = \dfrac{1}{x}$ and attempting to use $u = x^2 + 1$ and $\dfrac{dv}{dx} = \ln x$ in the formula

$$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx.$$

Candidates who correctly gave the intermediate result $\left[\left(\dfrac{x^3}{3} + x \right) \ln x \right]_1^e - \displaystyle\int_1^e \left(\dfrac{x^3}{3} + x \right) \dfrac{1}{x} \, dx$

often failed to use a bracket for the second part of the expression when they integrated and went on to make a sign error by giving $-\dfrac{x^3}{9} + x$ rather than $-\dfrac{x^3}{9} - x$.

12. Part (a)(i) was generally well done. In part (a)(ii), many had difficulty in differentiating $\cos(2x^3)$ and $-\sin(6x^2)$ was commonly seen. When the quotient rule was applied, it was often very unclear if candidates were using a correct version of the rule and candidates should be encouraged to quote formulae they are using. Notational carelessness often loses marks in questions of this kind. If $\cos(2x^3)$ is differentiated and the expression $6x^2 - \sin(2x^3)$ results, the examiner cannot interpret this as $6x^2 \times (-\sin(2x^3))$ unless there is some evidence that the candidate interprets it this way. A substantial proportion of those who wrote down $6x^2 - \sin(2x^3)$ showed in their later work that it had been misinterpreted. Similarly in the denominator of the quotient rule $3x^2$, as opposed to $(3x)^2$ or $9x^2$, cannot be awarded the appropriate accuracy mark unless a correct expression appears at some point. The use of the product rule in such questions is a disadvantage to all but the ablest candidates. In this case, few who attempted the question this way could handle the 3 correctly and the negative indices defeated many.

Part (b) was clearly unexpected by many candidates and some very lengthy attempts began by expanding $\sin(2y + 6)$ as $\sin 2y \cos 6 + \cos 2y \sin 6$. This led to attempts to use the product rule and errors like $\dfrac{d}{dx}(\sin 6) = \cos 6$ were frequent. Many could, however, get the first step $\dfrac{dx}{dy} = 8\cos(2y + 6)$ but on inverting to get $\dfrac{dx}{dy}$ simply turned the y into an x. Those reaching the correct $\dfrac{dx}{dy} = \dfrac{1}{\cos(2y + 6)}$ usually stopped there and the correct solution, in terms of x, was achieved by less than 10% of candidates. The answer $\dfrac{1}{8\cos\left(\arcsin\left(\frac{x}{4}\right)\right)}$ was accepted for full marks.

13. (a)   (i)   Most candidates demonstrated good knowledge of trigonometric differentiation but there were a number of errors particularly in the derivative of sec 2x.

      (ii)   Candidates did not always apply the function of a function rule, but the most common error seen in this part was d/dx(ln2x) = 1/2x or 2/x. Expanding using the binomial theorem prior to differentiation was rarely seen and when used, often contained inaccuracies.

  (b)   Knowledge of product and quotient rules was good but execution sometimes poor. There was a lack of sustained accuracy in algebra manipulation and much alteration to obtain the answer on the paper. Most candidates did not factorise out the (x-1) factor until the last line of the solution. There were however a few excellent solutions using the division method. Many replaced solutions to this part were inserted later in the answer book, and candidates are advised to make clear reference to such replaced solutions (with a page reference) in their original solution.

14. No Report available for this question.